Journal of Engineering Physics and Thermophysics, Vol. 83, No. 2, 2010

HEAT CONDUCTION AND HEAT TRANSFER IN TECHNOLOGICAL PROCESSES

MODELING OF HEAT CONDUCTION PROCESSES IN LAYERED HYBRID COMPOSITES OF REGULAR STRUCTURE WITH SLIT-LIKE LAYERS

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Models of the thermal conductivity of a layered medium with regularly alternating slit-like layers without regard for the time of relaxation of materials of the composition phases and with due regard for this relaxation time are suggested. It is shown that the effective thermal conductivity coefficients of such a medium depend substantially on the composition temperature. Concrete calculations of the effective characteristics of layered compositions depending on temperature, transverse dimension of slit-like layers, and on their specific volume content are carried out.

Keywords: *heat conduction, hybrid composites, slit-like layers, generalized Fourier law, Stefan–Boltzmann law, relaxation time.*

Introduction. In modern power plants, jet engines of aerospace technology, and laser, and MHD devices, extensive use is made of thin-walled elements of the type of shells and plates for efficient thermal insulation or accumulation and transfer of heat and provision of reliable operation of plants and engines at elevated levels of temperatures and power loadings. The use of homogeneous structure materials in such plants has virtually exhausted the potentialities of their improvement. Further breakthrough is possible along the path of creating layered composite structures by synthesizing various materials that could ensure discrete, continuous, or discrete-continuous change in the thermophysical and mechanical characteristics of a product. Here, some of the layers can be made in the form of slits ensuring a high level of thermal insulation in a transverse direction. From the technological point of view, the creation of such layered structures presents no serious problems, and they are finding a wide application as efficient carrying elements of transport and power plants, as well as of the elements of aviation-space technology.

The present investigation is devoted to the construction of models of thermal conductivity of layered hybrid composites of regular structure in the presence of slit-like layers in them.

Thermophysical Model of a Layered Medium with Regularly Alternating Slit-Like Layers without Account for the Time of Relaxation of the Materials of Composition Phases. We will consider a layered hybrid composite with regularly alternating (in the direction of x_3) thin anisotropic layers among which slit-like layers recur regularly. At an arbitrary point $\mathbf{x} = (x_1, x_2, x_3)$ we will isolate a small representative element of the volume $\Delta V = \Delta x_1 \times \Delta x_2 \times \Delta x_3$. Since it is very difficult to establish the actual distribution of heat fluxes and of the temperature field in a layered composite of the indicated structure, then in finding practically applicable dependences for determining effective thermophysical characteristics in the form of the components of linear heat conduction, some assumptions should be made similar to those used by the author in [1] to derive formulas that determine the effective thermal conductivity coefficients of a layered-fibrous medium that agree well with experiment [2]:

1. Within the limits of the representative element isolated from a composite at a minilevel, all the layers are homogeneous and anisotropic, have a constant thickness, and are parallel to the plane (x_1, x_2), therefore the layered material represents a macroscopically quasi-continuous anisotropic body.

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2. The relationship between the heat flux vector and temperature gradient in all solid phases of the composition obeys the Fourier linear heat conduction law; heat fluxes in slit-like layers are determined by the Stefan–Boltzmann law.

3. At the boundaries between regularly alternating solid layers the conditions of an ideal heat contact are realized; at the boundaries between the solid and slit-like layers the condition of contact via a heat flux is realized.

4. The increment in the averaged temperature T along an arbitrarily oriented segment of elementary length Δl is equal to the sum of the increments of temperatures in the phases of the composition that cross this segment.

5. The average heat flux through an arbitrarily oriented elementary area is counted using the rule of a simple mixture of heat fluxes in the phases of the composite.

The further line of reasoning is the same as in [1]; therefore, to save space, we will restrict ourselves to a brief account.

For the convenience of further presentation we will introduce the following idealization into consideration. We will assume that the slit-like layers are filled with a certain fictitious material; the relationship between the heat fluxes and the temperature gradient in this fictitious material does not obey the Fourier law but is defined by relations obtained in the process of reasoning on the basis of the Stefan–Boltzmann law. At the boundaries between the fictitious material and neighboring solid layers the conditions of an ideal thermal contact between the real and fictitious materials are fulfilled.

The following normalization condition is valid:

$$\Omega + \sum_k \omega_k = 1 . \tag{1}$$

Here and below summation is carried out over the indicated index from 1 to K if the limits are not set. If $\Omega = 0$, we obtain the case of a layered composite without slit-like layers.

According to the fifth assumption, for the components of the averaged heat flux vector we have

$$q_i = \Omega Q_i + \sum_k \omega_k q_i^{(k)} , \quad i = 1, 2, 3 . \tag{2}$$

From the fourth assumption, with regard for the adopted idealization, by analogy with [1], the following equalities result:

$$\partial_i T = \Omega \partial_i \theta + \sum_k \omega_k \partial_i T_k , \quad i = 1, 2, 3 . \tag{3}$$

According to the third hypothesis, with allowance for the adopted idealization, within the limits of the representative element the equalities resulting from the conjugation conditions hold:

$$\partial_i T_k = \partial_i \theta , \quad 1 \le k \le K , \quad i = 1, 2 ; \tag{4}$$

$$q_3^{(k)} = Q_3 , \quad 1 \le k \le K . \tag{5}$$

Having substituted equalities (4) and (5) into relations (2) and (3), subject to (1), we obtain

$$\partial_i T_k = \partial_i \theta = \partial_i T , \quad 1 \le k \le K , \quad i = 1, 2 ; \tag{6}$$

$$q_3^{(k)} = Q_3 = q_3 , \quad 1 \le k \le K . \tag{7}$$

By virtue of the second assumption the following relations hold:

$$q_i^{(k)} = -\sum_{j=1}^{3} \lambda_{ij}^{(k)} \partial_j T_k \,, \quad i = 1, 2, 3 \,; \tag{8}$$

$$Q_3 = -\,\varepsilon\sigma \left[(\theta + \Delta_3\theta)^4 - \theta^4 \right], \tag{9}$$

and moreover at the considered minilevel of the representative element

$$\Delta_3\theta = d\partial_3\theta + O\,(d^2) \approx d\partial_3\theta \,, \tag{10}$$

since the dimension d relates to the microlevel, i.e., $d << \Delta x_3$.

Substituting (10) into (9) and, as before, neglecting terms of order $O(d^2)$, we will have

$$Q_3 = -D\theta^3 \partial_3\theta \,, \quad D = 4\varepsilon\sigma d = \text{const} \,. \tag{11}$$

Relation (11) determines the law that connects the heat flux Q_3 with the derivative $\partial_3\theta$ in the fictitious material that fills the slit-like layers.

The right-hand side of equality (11) depends on the fictitious temperature θ. This is associated with certain inconvenience from the viewpoint of the final form of the governing heat conduction equations for the composite medium as a whole that must contain only the averaged temperature T of the composition. Therefore we will calculate the average temperature of the composition within the representative element:

$$\frac{1}{\Delta V} \iiint\limits_{\Delta V} T dV = \frac{1}{\Delta V} \iiint\limits_{\Delta V} \left(\Omega\theta + \sum_k \omega_k T_k \right) dV \,, \quad dV = dx_1 dx_2 dx_3 \,. \tag{12}$$

Taking into account the conditions of the conjugation of temperatures at the boundaries of the contact of different phases of the composition, from Eq. (12), discarding terms of order $O(\Delta x_i)$, on the basis of the integral mean-value theorem, we obtain

$$\theta = T \,. \tag{13}$$

Subject to (13), Eq. (11) finally yields

$$Q_3 = -\,DT^3 \partial_3\theta \,. \tag{14}$$

We will substitute relations (8) and (14) into Eqs. (2), then, subject to (6) and (7), we will obtain

$$q_i = \Omega Q_i - \sum_{j=1}^{2} \sum_k \omega_k \lambda_{ij}^{(k)} \partial_j T - \sum_k \omega_k \lambda_{i3}^{(k)} \partial_3 T_k \,, \quad i = 1, 2 \,; \tag{15}$$

$$q_3 = -\,DT^3 \partial_3\theta \,. \tag{16}$$

After the substitution of relations (8) and (14) into the conjugation conditions (5), with allowance for (6), we will have the equalities

$$\sum_{j=1}^{2} \lambda_{3j}^{(k)} \partial_j T + \lambda_{33}^{(k)} \partial_3 T_k = DT^3 \partial_3\theta \,, \quad k = 1, 2, ..., K \,,$$

whence

$$\partial_3 T_k = \frac{DT^3}{\lambda_{33}^{(k)}} \partial_3 \theta - \sum_{j=1}^{2} \frac{\lambda_{3j}^{(k)}}{\lambda_{33}^{(k)}} \partial_j T , \quad k = 1, 2, ..., K . \tag{17}$$

Subject to (17) at $i = 3$, Eq. (3) yields

$$\partial_3 T = A\,(T)\,\partial_3 \theta - \sum_{j=1}^{2} \sum_{k} \frac{\omega_k \lambda_{3j}^{(k)}}{\lambda_{33}^{(k)}} \partial_j T , \tag{18}$$

where

$$A\,(T) = \Omega + \frac{DT^3 \sum\limits_{k} \omega_k}{\lambda_{33}^{(k)}} . \tag{19}$$

From Eq. (18) we have

$$\partial_3 \theta = \frac{1}{A\,(T)} \left(\partial_3 T + \sum_{j=1}^{2} \sum_{k} \frac{\omega_k \lambda_{3j}^{(k)}}{\lambda_{33}^{(k)}} \partial_j T \right). \tag{20}$$

Having substituted Eq. (20) into equality (16), we obtain

$$q_3 = - \frac{DT^3}{A\,(T)} \left(\partial_3 T + \sum_{j=1}^{2} \sum_{k} \frac{\omega_k \lambda_{3j}^{(k)}}{\lambda_{33}^{(k)}} \partial_j T \right). \tag{21}$$

At the same time, from equalities (15), subject to (17) and (20), it follows that

$$q_i = \Omega Q_i - \sum_{j=1}^{2} \sum_{k} \omega_k \lambda_{ij}^{(k)} \partial_j T - \sum_{k} \omega_k \frac{\lambda_{i3}^{(k)}}{\lambda_{33}^{(k)}} \left[\frac{DT^3}{A\,(T)} \left(\partial_3 T + \sum_{j=1}^{2} \sum_{n} \frac{\omega_n \lambda_{3j}^{(n)}}{\lambda_{33}^{(n)}} \partial_j T \right) - \sum_{j=1}^{2} \lambda_{3j}^{(k)} \partial_j T \right], \quad i = 1, 2 , \tag{22}$$

where the longitudinal heat fluxes Q_i inside slit-like layers can be determined, for example, from the Stefan–Boltzmann law:

$$Q_i = - \sigma \left[\varepsilon_i^{(+)} \left(T_i^{(+)} \right)^4 - \varepsilon_i^{(-)} \left(T_i^{(-)} \right)^4 \right], \quad i = 1, 2 . \tag{23}$$

According to Eq. (23), the heat fluxes Q_i in (22) can be considered known (given) functions. The coefficients

$$\Lambda_{ij}\,(T) = \sum_{k} \omega_k \left(\lambda_{ij}^{(k)} - \frac{\lambda_{i3}^{(k)} \lambda_{3j}^{(k)}}{\lambda_{33}^{(k)}} \right) + \frac{DT^3}{A\,(T)} \sum_{k} \frac{\omega_k \lambda_{i3}^{(k)}}{\lambda_{33}^{(k)}} \sum_{n} \frac{\omega_n \lambda_{3j}^{(n)}}{\lambda_{33}^{(n)}} ,$$

$$\Lambda_{i3}\,(T) = \Lambda_{3i}\,(T) = \frac{DT^3}{A\,(T)} \sum_{k} \frac{\omega_k \lambda_{i3}^{(k)}}{\lambda_{33}^{(k)}} , \quad \Lambda_{33}\,(T) = \frac{DT^3}{A\,(T)} , \quad i, j = 1, 2 , \tag{24}$$

entering into equalities (21) and (22) can be interpreted as the effective heat conduction coefficients of the considered layered hybrid composite whose governing equations correspond to the following generalized Fourier law:

$$q_i = -\sum_{j=1}^{3} \Lambda_{ij}(T)\,\partial_j T + q_i^0\,, \quad i = 1, 2\,, \quad q_3 = -\sum_{j=1}^{3} \Lambda_{3j}(T)\,\partial_j T\,, \tag{25}$$

where $q_i^0 = \Omega Q_i$ are the given heat fluxes.

By virtue of equalities (19) and (24), the coefficients Λ_{ij} depend on the temperature T of the composition; therefore such a composite material should be considered as thermally sensitive even in the case where the thermal sensitivity of the materials of the composition phases is not taken into account (the dependence of the coefficients $\lambda_{ij}^{(k)}$ on T is not considered).

R e m a r k 1. An important feature of the model constructed is the possibility of determining the averaged temperature $\partial_i T$ of heat fluxes by the gradient and of the gradients of temperatures $\partial_i T_k$ in all of the composition phases. Indeed, if from the solution of the heat conduction problem for the considered composite medium the derivatives of averaged temperature $\partial_i T$ are known, then from Eq. (20) it is possible to determine the derivative $\partial_3 \theta$ of the fictitious temperature and then, from Eqs. (6) and (17), to determine the temperature gradients in all the layers $\partial_i T_k$ $(1 \le k \le K,\ i = 1,\ 2,\ 3)$ and then, using the Fourier law (8), to determine heat fluxes in the composition phases.

If in the composite considered slit-like layers are absent, then from Eqs. (19), (24), and (25) at $\Omega = 0$ we obtain

$$q_i = -\sum_{j=1}^{3} \Lambda_{ij}\partial_j T\,, \quad i = 1, 2, 3\,; \tag{26}$$

$$\Lambda_{ij} = \sum_k \omega_k \left(\lambda_{ij}^{(k)} - \frac{\lambda_{i3}^{(k)}\lambda_{3j}^{(k)}}{\lambda_{33}^{(k)}} \right) + \sum_k \frac{\omega_k \lambda_{i3}^{(k)}}{\lambda_{33}^{(k)}} \sum_n \frac{\omega_n \lambda_{3j}^{(n)}}{\lambda_{33}^{(n)}} \left(\sum_m \frac{\omega_m}{\lambda_{33}^{(m)}} \right)^{-1},$$

$$\Lambda_{i3} = \Lambda_{3i} = \sum_k \frac{\omega_k \lambda_{i3}^{(k)}}{\lambda_{33}^{(k)}} \left(\sum_n \frac{\omega_n}{\lambda_{33}^{(n)}} \right)^{-1}, \quad \Lambda_{33} = \left(\sum_k \frac{\omega_k}{\lambda_{33}^{(k)}} \right)^{-1}, \quad i, j = 1, 2\,, \quad \Omega = 0\,. \tag{27}$$

Relations (26), subject to (27), determine the Fourier law for a layered composite, at the boundaries between the anisotropy layers of which the conditions of an ideal thermal contact are fulfilled.

If in each solid layer one of the principal axes of the material anisotropy coincides with the x_3 axis ($\lambda_{i3}^{(k)} = 0$, $i = 1, 2$), then Eq. (24) yields

$$\Lambda_{ij} = \sum_k \omega_k \lambda_{ij}^{(k)}\,, \quad \Lambda_{i3} = \Lambda_{3i} = 0\,, \quad \Lambda_{33}(T) = \frac{DT^3}{A(T)}\,,$$

$$\lambda_{i3}^{(n)} = 0\,, \quad i, j = 1, 2\,, \quad 1 \le n \le K\,, \ \Omega > 0\,, \tag{28}$$

and from Eq. (27) it follows that

$$\Lambda_{ij} = \sum_k \omega_k \lambda_{ij}^{(k)}\,, \quad \Lambda_{i3} = \Lambda_{3i} = 0\,, \quad \Lambda_{33}^{-1} = \sum_k \omega_k / \lambda_{33}^{(k)}\,,$$

$$\lambda_{i3}^{(n)} = 0\,, \quad i, j = 1, 2\,, \quad 1 \le n \le K\,, \ \Omega = 0\,. \tag{29}$$

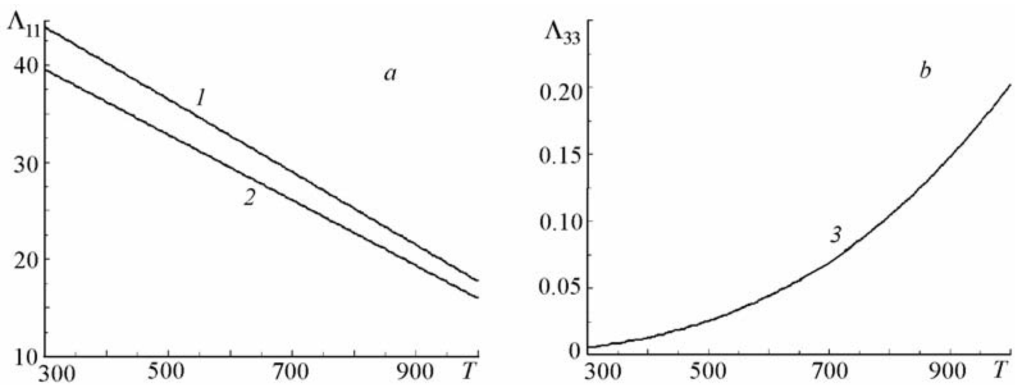
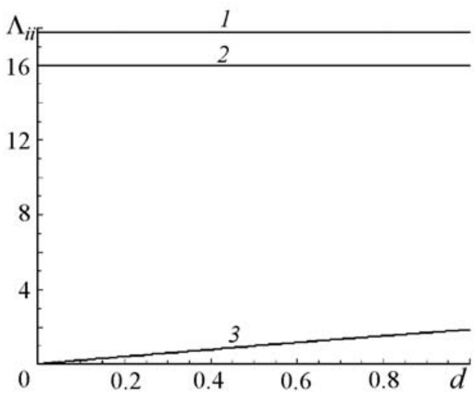


Fig. 1. Effective coefficients of a layered composite in the longitudinal (a) and transverse (b) directions vs. the composite temperature. Λ_{ii}, W/(m·K); T, K.



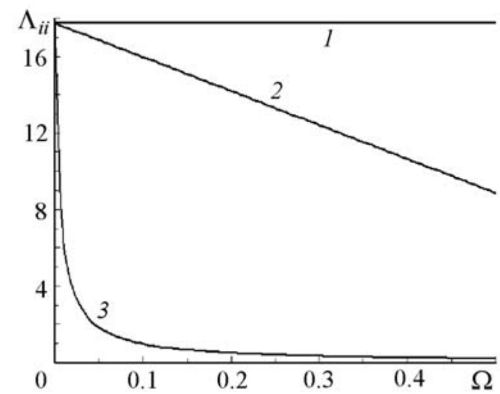


Fig. 2. Effective coefficients of a layered composite vs. the transverse dimension of slit-like layers. Λ_{ii}, W/(m·K); d, m.

Fig. 3. Effective coefficients of a layered composite vs. the specific volume content of slit-like layers. Λ_{ii}, W/(m·K).

In equalities (28) and (29) the effective coefficients of longitudinal heat conduction Λ_{ij} (i, j = 1, 2) are determined by the rule of a simple mixture; it is known [3] that the predicted values calculated by this formula agree well with the experiment. At the same time, the formula for the effective coefficient of transverse heat conduction Λ_{33} in (29) coincides with an analogous formula obtained by the author in [1] for a layered reinforced medium, which also agrees well with experimental data [2].

As an example we will calculate the effective thermal conductivity coefficients of a layered composite obtained by regular alteration of slit-like and iron layers (K = 1). For iron the dependence of the thermal conductivity coefficients on temperature can be given in the first approximation in the form [4, 5]

$$\lambda_{ij}^{(1)} = 45\delta_{ij}\left[1 - \frac{T - 273}{1200}\right], \quad \text{W/(m·K)}\,, \quad i, j = 1, 2, 3\,.$$

According to relations (1), (19), and (28), in this case we have

$$\Lambda_{11} = \Lambda_{22} = \lambda_{11}^{(1)}\omega_1\,, \quad \Lambda_{33} = DT^3 \Big/ \left(\Omega + \frac{DT^3\omega_1}{\lambda_{11}^{(1)}}\right), \quad \omega_1 = 1 - \Omega\,, \quad \Lambda_{ij} = 0\,, \quad i \neq j\,.$$

Figures 1–3 present the dependences of Λ_{ii} of the considered composite on the temperature T, transverse dimension d of slit-like layers, and the specific volume content Ω of these layers in the composition. The curves 1 in these figures are given for comparison and correspond to an isotopic material, iron; curves 2 characterize the effective longitudinal thermal conductivity coefficients of the composition ($\Lambda_{11} = \Lambda_{22}$), and curves 3 the transverse (Λ_{33}) thermal conductivity coefficients of the composition. The curves in Fig. 1 were calculated at $\Omega = 0.1$, $d = 0.1$ mm; in Fig. 2, at $T = 1000$ K, $\Omega = 0.1$; in Fig. 3, at $T = 1000$ K and $d = 0.5$ mm. In all the calculations it was adopted that $\varepsilon = 0.9$.

A comparison of curves 2 in Fig. 1a and of curves 3 in Fig. 1b shows that in this case (at the given structural parameters of the composition) the effective coefficient of transverse thermal conductivity is negligibly small as compared to the coefficient of the longitudinal thermal conductivity of the composition ($\Lambda_{33} << \Lambda_{11} = \Lambda_{22}$). However, as is seen from the comparison of curves 2 and 3 in Figs. 2 and 3, under certain conditions (T) and values of the structural parameters of the composition (d, Ω) the coefficients $\Lambda_{11} = \Lambda_{22}$ and Λ_{33} can be commensurable in their values, and in these cases one cannot already neglect the transverse thermal conductivity of the composition in comparison with its longitudinal one. It should be noted that if on the basis of assumptions 1–5 we construct a model of the thermal conductivity of a layered composition neglecting heat transfer in slit-like layers occurring according to the Stefan–Boltzmann law, then as a result we will obtain that the effective coefficient of transverse thermal conductivity (Λ_{33}) is identically equal to zero. This result does not agree with the behavior of curves 3 in Figs. 2 and 3.

A Thermophysical Model of a Layered Medium with Regularly Alternating Slit-Like Layers with Account for the Time of Relaxation of the Materials of the Composition Phases. We will consider the former composite material and will use assumptions 1–5 but we will take into account that in the case of highly intensive processes of heat transfer the relations of the generalized Fourier law for all solid phases of the composite differ from (8), and in the coordinate system $x_i^{(k)}$ connected with the principal axes of the thermophysical anisotropy of the kth layer they have the form

$$\overline{q}_i^{(k)} + \tau_i^{(k)} \partial_t \overline{q}_i^{(k)} = -\overline{\lambda}_{ii}^{(k)} \partial_i^{(k)} T_k \,, \quad 1 \le k \le K \,, \quad i = 1, 2, 3 \,. \tag{30}$$

Next, for simplicity of presentation, it is assumed that one of the principal axes of the anisotropy of each layer coincides with the transverse direction, i.e., $x_3^{(k)} = x_3$, $\lambda_{i3}^{(k)} = 0$, $\overline{\lambda}_{33}^{(k)} = \lambda_{33}^{(k)}$, $1 \le k \le K$, $i = 1, 2$. Here the following relations are valid:

$$q_1^{(k)} = n_1^{(k)} \overline{q}_1^{(k)} - n_2^{(k)} \overline{q}_2^{(k)} \,, \quad q_2^{(k)} = n_2^{(k)} \overline{q}_1^{(k)} + n_1^{(k)} \overline{q}_2^{(k)} \,, \quad q_3^{(k)} = \overline{q}_3^{(k)} \,; \tag{31}$$

$$\partial_1^{(k)} T_k = n_1^{(k)} \partial_1 T_k + n_2^{(k)} \partial_2 T_k \,, \quad \partial_2^{(k)} T_k = -n_2^{(k)} \partial_1 T_k + n_1^{(k)} \partial_2 T_k \,,$$

$$\partial_3^{(k)} T_k = \partial_3 T_k \,, \quad n_1^{(k)2} + n_2^{(k)2} = 1 \,, \quad n_3^{(k)} = 0 \,, \quad 1 \le k \le K \,. \tag{32}$$

We will integrate relations (30) over time; then

$$\overline{q}_i^{(k)} (\mathbf{x}, t) = \left[\overline{q}_{i0}^{(k)} (\mathbf{x}) - \int_{t_0}^{t} \frac{\overline{\lambda}_{ii}^{(k)}}{\tau_i^{(k)}} \exp \left(\frac{s}{\tau_i^{(k)}} \right) \partial_i^{(k)} T_k (\mathbf{x}, s) \, ds \right] \exp \left(- \frac{t - t_0}{\tau_i^{(k)}} \right),$$

$$\mathbf{x} = \{x_1, x_2, x_3\} \,, \quad \overline{q}_{i0}^{(k)} (\mathbf{x}) = \overline{q}_i^{(k)} (\mathbf{x}, t_0) \,, \quad 1 \le k \le K \,, \quad i = 1, 2, 3 \,. \tag{33}$$

Below we will assume that $t_0 = 0$.

According to assumptions 3–5 (with allowance for the earlier idealization), equalities (2)–(7) and (16) remain valid. From conjugation condition (5), subject to (14), (31), (33), we have

$$- DT^3 \partial_3 \theta = \left[q_{30}^{(k)} (\mathbf{x}) - \int_0^t \frac{\lambda_{33}^{(k)}}{\tau_3^{(k)}} \exp \left(\frac{s}{\tau_3^{(k)}} \right) \partial_3 T_k (\mathbf{x}, s) \, ds \right] \exp \left(- \frac{t}{\tau_3^{(k)}} \right), \quad 1 \le k \le K \,. \tag{34}$$

From Eq. (34) we express the derivative $\partial_3 T_k(\mathbf{x}, t)$ in terms of $\partial_3\theta(\mathbf{x}, t)$, which after elementary transformations yields

$$\partial_3 T_k(\mathbf{x}, t) = \frac{D\tau_3^{(k)}}{\lambda_{33}^{(k)}}\left[\frac{T^3}{\tau_3^{(k)}}\partial_3\theta + \partial_t(T^3\partial_3\theta)\right], \quad 1 \le k \le K. \tag{35}$$

We substitute derivatives (35) into equality (3) at $i = 3$, after which we obtain (see Eq. (19))

$$\partial_3 T(\mathbf{x}, t) = \frac{A(T)}{T^3}(T^3\partial_3\theta(\mathbf{x}, t)) + D\sum_k \frac{\omega_k\tau_3^{(k)}}{\lambda_{33}^{(k)}}\partial_t(T^3\partial_3\theta(\mathbf{x}, t)). \tag{36}$$

Integrating Eq. (36) over time, we determine the dependence of $T^3\partial_3\theta$ on the derivative $\partial_3 T$; after this, on the basis of Eq. (16), we will have

$$q_3(\mathbf{x}, t) = \exp\left(-\int_0^t \frac{A(T(\mathbf{x}, \xi))\, d\xi}{DT^3(\mathbf{x}, \xi)\sum_k \omega_k\tau_3^{(k)}/\lambda_{33}^{(k)}}\right)$$

$$\times\left[q_{30}(\mathbf{x}) - \int_0^t \frac{\partial_3 T(\mathbf{x}, s)}{\sum_k \omega_k\tau_3^{(k)}/\lambda_{33}^{(k)}}\exp\left(\int_0^s \frac{A(T(\mathbf{x}, \xi))\, d\xi}{DT^3(\mathbf{x}, \xi)\sum_k \omega_k\tau_3^{(k)}/\lambda_{33}^{(k)}}\right)ds\right], \quad q_{30}(\mathbf{x}) = q_3(\mathbf{x}, 0). \tag{37}$$

We substitute relations (33) into (31) and the latter into (2); then with allowance for (32) and (6) we will obtain

$$q_1(\mathbf{x}, t) = \Omega Q_1 + \sum_k \omega_k n_1^{(k)}\left[\bar{q}_{10}^{(k)}(\mathbf{x}) - \int_0^t \frac{\bar{\lambda}_{11}^{(k)}}{\tau_1^{(k)}}\exp\left(\frac{s}{\tau_1^{(k)}}\right)\left(n_1^{(k)}\partial_1 T(\mathbf{x}, s) + n_2^{(k)}\partial_2 T(\mathbf{x}, s)\right)ds\right]$$

$$\times\exp\left(-\frac{t}{\tau_1^{(k)}}\right) - \sum_k \omega_k n_2^{(k)}\left[\bar{q}_{20}^{(k)}(\mathbf{x}) - \int_0^t \frac{\bar{\lambda}_{22}^{(k)}}{\tau_2^{(k)}}\exp\left(\frac{s}{\tau_2^{(k)}}\right)\left(-n_2^{(k)}\partial_1 T(\mathbf{x}, s) + n_1^{(k)}\partial_2 T(\mathbf{x}, s)\right)ds\right]\exp\left(-\frac{t}{\tau_2^{(k)}}\right), \tag{38}$$

$$q_2(\mathbf{x}, t) = \Omega Q_2 + \sum_k \omega_k n_2^{(k)}\left[\bar{q}_{10}^{(k)}(\mathbf{x}) - \int_0^t \frac{\bar{\lambda}_{11}^{(k)}}{\tau_1^{(k)}}\exp\left(\frac{s}{\tau_1^{(k)}}\right)\left(n_1^{(k)}\partial_1 T(\mathbf{x}, s) + n_2^{(k)}\partial_2 T(\mathbf{x}, s)\right)ds\right]$$

$$\times\exp\left(-\frac{t}{\tau_1^{(k)}}\right) + \sum_k \omega_k n_1^{(k)}\left[\bar{q}_{20}^{(k)}(\mathbf{x}) - \int_0^t \frac{\bar{\lambda}_{22}^{(k)}}{\tau_2^{(k)}}\exp\left(\frac{s}{\tau_2^{(k)}}\right)\left(-n_2^{(k)}\partial_1 T(\mathbf{x}, s) + n_1^{(k)}\partial_2 T(\mathbf{x}, s)\right)ds\right]\exp\left(-\frac{t}{\tau_2^{(k)}}\right),$$

where

$$\bar{q}_{10}^{(k)}(\mathbf{x}) = n_1^{(k)}q_{10}^{(k)}(\mathbf{x}) + n_2^{(k)}q_{20}^{(k)}(\mathbf{x}), \quad \bar{q}_{20}^{(k)}(\mathbf{x}) = -n_2^{(k)}q_{10}^{(k)}(\mathbf{x}) + n_1^{(k)}q_{20}^{(k)}(\mathbf{x}),$$

$$q_{i0}^{(k)}(\mathbf{x}) = q_i^{(k)}(\mathbf{x}, 0), \quad i = 1, 2, \quad 1 \le k \le K. \tag{39}$$

Relations (37) and (38), with allowance for equalities (19) and (39), form the sought system of governing heat conduction equations ($q_i = f_i(\partial_j T)$ of the layered hybrid composite material with slit-like layers with account for the relaxation time of the composition phases.

R e m a r k 2. If up to the initial moment $t = t_0 = 0$ there was a stationary temperature field, then, knowing $\partial_i T(\mathbf{x}, 0)$, according to Remark 1, we may determine $q_{i0}^{(k)}(\mathbf{x})$ entering into equalities (39) and thereby into (38), as well as the function $q_{30}(\mathbf{x})$ in equality (37).

Substituting relations (37) and (38) into the heat balance equation [6]:

$$\operatorname{div}\mathbf{q} + C\partial_t T + T\sum_{i=1}^{3}\sum_{j=1}^{3}\beta_{ij}\partial_t\varepsilon_{ij} = W\,, \quad \mathbf{q} = \{q_1, q_2, q_3\}\,, \tag{40}$$

we obtain a nonlinear integrodifferential heat conduction equation for the averaged temperature T of the composition, which will not be expounded here. The last group of terms on the left-hand side of Eq. (40) is taken into account if a coupled problem of thermal elasticity is considered.

For single-valued integration of the heat conduction equation (40), it should be supplemented with the well-known initial and boundary conditions of the theory of thermal conductivity of solids [6, 7], which will not be expounded here.

This work was carried out with financial support from the Russian Fundamental Research Foundation (code of the project 08-01-90403-Ukr_a) and from the Presidium of the Siberian Branch of the Russian Academy of Sciences (adhesion No. 10 of 15.01.09, project No. 72).

NOTATION

C, reduced heat capacity of composite, J·m^3/K; d, characteristic transverse dimension of slit-like layers, m; D, constant characterizing the transverse thermal conductivity of a fictitious material filling a slit-like layer, W/(m·K^4); K (≥ 1), number of the families of layers from solid materials; $n_i^{(k)}$, direction cosines of the local system of coordinates $x_i^{(k)}$; Q_i, components of the heat flux vector in slit-like layers in the directions x_i, W/m^2; q_i, components of the averaged heat flux vector in the global coordinate system x_i (i = 1, 2, 3), W/m^2; $q_i^{(k)}$, components of the heat flux vector in the layer of the kth family in the directions x_i, W/m^2; $\bar{q}_i^{(k)}$, components of the heat flux vector in the layer of the kth family in the directions $x_i^{(k)}$, W/m^2; $q_{i0}^{(k)}$, components of the heat flux vector in the layer of the kth family in the directions x_i at the initial moment, W/m^2; $\bar{q}_{i0}^{(k)}$, components of the heat flux vector in the layer of the kth family in the directions $x_i^{(k)}$ at the initial moment, W/m^2; T, average temperature of the composite, K; T_k, temperature of the layer of the kth family ($1 \leq k \leq K$), K; $T_i^{(-)}$ and $T_i^{(+)}$, temperatures of some radiating surfaces, external relative to the composite body, perpendicular to the directions x_i (i = 1, 2), K; t, time, sec; t_0, initial moment, sec; W, reduced power density of internal heat sources of the composition, W/kg; x_i (i = 1, 2, 3), global rectangular Cartesian coordinate system, m; $x_i^{(k)}$, local coordinate system connected with the principal axes of the thermophysical anisotropy of the material of the kth family layer, m; β_{ij}, components of the tensor of effective temperature rigidities of the composition, Pa/K; Δl, length of an elementary section arbitrarily oriented in space, m; ΔV, volume of the representative element, m^3; Δx_i (i = 1, 2, 3), linear dimensions of the representative element, m; $\Delta_3\theta$, temperature difference (in the direction of x_3) in the fictitious material filling the slit-like cavity, K; δ_{ij}, Kronecker symbol; ε, reduced emissivity of the surfaces bounding the slit-like layers; $\varepsilon_i^{(-)}$ and $\varepsilon_i^{(+)}$, emissivities of the surfaces with the temperatures $T_i^{(-)}$ and $T_i^{(+)}$; ε_{ij}, components of the tensor of elastic deformations; Λ_{ij}, effective coefficients of the thermal conductivity of the composite in the global coordinate system x_i, W/(m·K); $\lambda_{ij}^{(k)}$, coefficients of the thermal conductivity of the material of the layer of the kth family in the global coordinate system x_i, W/(m·K); $\bar{\lambda}_{ii}^{(k)}$, principal values of the tensor of the thermal conductivity coefficients of the kth family layer material in the local coordinate system $x_i^{(k)}$ ($\bar{\lambda}_{ij}^{(k)} = 0$, $i \neq j$), W/(m·K); θ, temperature of the fictitious material filling slit-like layers, K; $\sigma = 5.67\cdot10^{-8}$ W·m^{-2}·K^{-4}, Stefan constant; $\tau_i^{(k)}$, time of relaxation of the material of the kth family layer in the direction $x_i^{(k)}$ (i = 1, 2, 3), sec; Ω, specific volume content of slit-like layers; ω_k, specific volume content of the kth family layers ($1 \leq k \leq K$); ∂_i, operator of partial differentiation in the direction x_i; $\partial_i^{(k)}$, operator of partial differentiation in the direction $x_i^{(k)}$; ∂_t, operator of partial differentiation with respect to time.

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